Universitatea Transilvania din Brașov

InterdisciplinaryDoctoral School

ADMISSION TO DOCTORAL STUDIESSession September 2022

Field of doctoral studies: MATHEMATICS

Doctoral supervisor: Adela-Gabriela MIHAI

TOPICS FOR THE ADMISSION TO DOCTORAL STUDIES

TOPIC 1: *Statistical Structures in Riemannian and Pseudo-Riemannian Geometries*
Content / Main aspects to be considered: - Riemannian manifolds – fundamental notions. - Riemannian submanifolds; fundamental equations, fundamental theorems. - Pseudo-Riemannian manifolds – fundamental notions. - Pseudo-Riemannian submanifolds; fundamental equations, fundamental theorems. - Affine differential geometry – fundamental notions. - Dual affine connections. - Dual flat Riemannian structures. - Exponential families of probability distributions. - Geometry of the statistical models. - Statistical manifolds. - Submanifolds in statistical manifolds.
Recommended bibliography: 1. B.-Y. Chen, *Pseudo-Riemannian Geometry, δ-Invariants and Applications,* World Scientific, 2011. 2. S. Amari, *Differential-Geometric Methods in Statistics*, Springer, 1985. 3. S. Amari, *Information Geometry and Its Applications*, Springer, 2016. 4. A.-M. Li, Z. Hu, U. Simon, G. Zhao, *Global Affine Differential Geometry of Hypersurfaces*, De Gruyter, 2015.
Prerequisites / Remarks: *Basic notions on differential geometry, probability, statistics, mathematical analysis, linear algebra, differential equations are required.*

TOPIC 2: *Curvature Invariants of Riemannian Manifolds*
Content / Main aspects to be considered: - Riemannian manifolds – fundamental notions. - Submanifolds in Riemannian manifolds; fundamental formulae and equations. - Intrinsic and extrinsic invariants of a submanifold.

- Sectional curvature.
- Scalar curvature.
- Ricci curvature.
- δ-invariants and applications.
- Relations between extrinsic and intrinsic invariants of a submanifold.
- Obstructions to the existence of some special classes of submanifolds.
- Generalized curvatures.

Recommended bibliography:

1. M. Do Carmo, *Riemannian Geometry*, Birkhauser, 1992.
2. B.-Y. Chen, *Pseudo-Riemannian Geometry, δ-Invariants and Applications,* World Scientific, 2011.
3. B.-Y. Chen, *Total Mean Curvature and Submanifolds of Finite Type*, World Scientific, 2014.
4. J.-M. Morvan, *Generalized Curvatures*, Springer, 2008.

Prerequisites / Remarks: *Basic notions on differential geometry, mathematical analysis, linear algebra, differential equations are required.*

TOPIC 3: *Applications of Differential Geometry in Machine Learning*

Content / Main aspects to be considered:

- Differential geometry – fundamental notions.
- Riemannian manifolds – fundamental notions.
- Geometry of spaces of probabilistic models – fundamental notions.
- Geometry of statistical manifolds – fundamental notions.
- Diffusion kernels on statistical manifolds.
- Machine learning – fundamental notions.
- Geometric deep learning – fundamental notions.

Recommended bibliography:

1. S. Kobayashi, K. Nomizu, *Foundations of Differential Geometry*, vol I, II, Interscience Publication, Wiley, New York 1963, 1969.
2. M. Do Carmo, *Riemannian Geometry*, Birkhauser, 1992.
3. A. N. Gorban, B. Kégl, D. C. Wunsch, A. Zinovyev (Eds.), *Principal Manifolds for Data Visualization and Dimension Reduction*, Springer 2008.
4. Y. Ma, Y. Fu, *Manifold Learning Theory and Applications*, CRC Press, 2012.
5. S. Amari, *Information Geometry and Its Applications*, Springer, 2016.

Prerequisites / Remarks: *Basic notions on differential geometry, mathematical analysis, linear algebra, differential equations, machine learning are required.*

Doctoral supervisor,

Conf. Dr. Adela-Gabriela MIHAI

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Coordinator of the field of doctoral studies,

Prof. Dr. Radu Păltănea

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